A selective cell discarding scheme using packet-size information

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Abstract

In packet communications over an asynchronous transfer mode (ATM) network, if even one cell of a packet is discarded in the network, the entire packet becomes useless. However, other cells of the packet are still transmitted through the network which wastes network resources. This may decrease the packet throughput of users. We propose a selective cell discarding scheme that prevents such useless cells from being transmitted. In this scheme, the behavior of a virtual cell queue is simulated using packet-size information, and a newly arriving packet is accepted only if the length of this virtual queue is greater than or equal to a given threshold. The packet-size information is available when the connectionless network access protocol (CLNAP) and ATM adaptation layer (AAL) type 3/4 are used in the network. Computer simulations have shown that our scheme can prevent decreased packet throughput due to discarded cells, even when the traffic load is very high, and can fairly accept packets regardless of the packet sizes.

keywords: ATM network, connectionless service, selective cell discarding, queueing model, packet throughput

1 Introduction

In packet communications over an asynchronous transfer mode (ATM) network, each packet is divided into a number of cells at the source. The packet is then transmitted by cell-by-cell transmission in the ATM network and reassembled at the destination. It has been reported, though, that when buffer overflows occur in an ATM network because of traffic congestion, the packet throughput falls drastically for the following reasons [1, 2].

(a) If even one cell of a packet is lost in the ATM network, the packet cannot be reassembled at the destination: even if the packet is partially assembled, it will be discarded at an upper layer as an incomplete packet. Hence, successfully transmitted cells of the packet will be useless and the buffer space and link capacity used by them are wasted.

(b) In most cases of buffer overflow, several packets are being simultaneously transmitted through the buffer so cells of many packets are lost at the same time, making several packets useless.

(c) The flow control of the transmission control protocol (TCP) does not work well in some cases. When one packet is lost in the network, a version of the TCP flow control can quickly retransmit the packet to maintain packet throughput. However, when several packets are lost simultaneously, this mechanism may not work well. As a result, the size of the TCP congestion window is unnecessarily decreased and the value of the time-out timer is unnecessarily increased. Hence, the packet transmission rates of users become very low for a long time and the network resources are not used effectively.

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These phenomena occur because packets are transmitted by cell-by-cell transmission. However, if each packet is assembled and then divided into cells again at each switching node to achieve packet-based transmission, the transmission delay of the packet becomes longer and a lot of memory space is needed for the assembly. This would negate some of the merits of using ATM technology.

Hence, we propose a scheme of selective cell discarding that enables packet-based transmission without the need to assemble packets. In this scheme, we require that the size of a packet can be obtained from the contents of the first cell of the packet. Using this size information, we simulate the queueing behavior in the cell buffer as if all the cells of a packet arrive at the same time. A newly arriving packet is accepted if the length of the simulated queue is greater than or equal to a given threshold. Such packet size information is available when the connectionless network access protocol (CLNAP) [3] and ATM adaptation layer (AAL) type 3/4 [4] are used in the network. The queueing behavior can be implemented by using a cell counter.

In Section 2, we explain the broadband connectionless data bearer service, and in Section 3, we review existing schemes of selective cell discarding and describe our scheme. The performance characteristics of our scheme are evaluated in Section 4, and are compared to those of existing schemes in terms of reasons (a) and (b) for throughput decrease mentioned above. This comparison was made by computer simulation, where a Monte Carlo method and actual LAN traffic data were used to generate packet arrivals. Our results show that our scheme can prevent packet throughputs from decreasing under various traffic conditions, and can also accept packets fairly regardless of their lengths.

2 Broadband connectionless data bearer service

Broadband connectionless data bearer service [3, 5] provides data communications over ATM networks without connection setup procedures such as connection acceptance control. It also makes it possible for switching nodes to transmit packets by cell-by-cell transmission without assembling them. These advantages enable high-speed packet communications because no connection setup time, which is needed for connection-oriented communications, or time for assembling packets, which is needed for packet-based data communications, is necessary [6, 7, 8]. However, the absence of connection acceptance control also means that traffic congestion at a link cannot be prevented in advance; if such congestion occurs, the packet throughput may decrease due to the phenomena mentioned in Section 1.

Figure 1 shows a broadband connectionless service network, and Fig. 2 shows the protocol formats of the CLNAP and AAL type 3/4 [4, 3]. In this network, routers and some hosts are directly connected to connectionless servers (CLSs) via ATM permanent virtual channels (PVCs), and CLSs are also connected to each other via ATM PVCs. In such routers or hosts, user data such as an IP packet is put into the payload of a CLNAP protocol data unit (PDU), and the CLNAP-PDU is put into the payload of a common part convergence sublayer (CPCS) PDU. This CPCS-PDU is divided into segmentation and reassembly (SAR) PDUs with a 48-byte data length including a 4-byte header, and the SAR-PDUs are put into the payloads of cells. The position (beginning/ continuation/ end/ single) of a cell in the CPCS-PDU can be identified by using the value of the segment type (ST) in the SAR header of the cell. If a cell arriving at a CLS is the beginning cell of a CPCS-PDU, the CLS reads the destination address (DA) from the cell, decides the output link to which it is sent, replaces the values of the virtual path indicator (VPI), virtual channel indicator (VCI), multiplexing identification (MID) of it with new values corresponding to the output link, and sends it to the output link. Other cells of the same CPCS-PDU are also sent to the same output link that the beginning cell is sent to by replacing the values of their VPI, VCI, and MID with the same new values. The CLS can, therefore, identify each CPCS-PDU using only the values of the VPIs, VCIs, MIDs, and STs of cells, and get the size of a CPCS-PDU from the value of the buffer allocation size (BAsize) written in the beginning cell of the CPCS-PDU at the arrival time of the cell. Hereafter, we will call a CPCS-PDU a packet, with its size represented by the number of cells. We emphasize, though, that our proposal can be applied to other types of ATM-based data communication networks if each packet can be identified by using only the information in cells and the size of a packet can be obtained at the arrival time of its beginning cell.
3 Selective cell discarding

3.1 Selective cell discarding schemes

In an ATM-based switching node, such as a CLS and an ATM switch, arriving cells are usually stored in the output buffer according to a first-in, first-out (FIFO) rule, and a cell arriving when the output buffer is full is simply discarded (buffer overflow). On the other hand, a selective cell discarding scheme discards cells according to a certain rule even if the output buffer is not full. The aims of such a scheme are to prevent the phenomena mentioned in Section 1 from occurring and to maintain high packet throughput.

Figure 3 shows a general model of selective cell discarding schemes, which can be categorized into two types. One is the reactive type, where actions are taken after a buffer overflow occurs. The other is the preventive type, where acceptance of an arriving packet is decided when the beginning cell of the packet arrives; if the packet is rejected, then all of its cells are discarded. Of course, it is also possible to combine both types of scheme.

A typical reactive-type scheme is partial packet discarding (PPD) [6]; if a cell of a packet is discarded because of a buffer overflow, then the other cells of the packet arriving after that cell are discarded. However, as we discuss in Section 5, PPD cannot effectively prevent the phenomena mentioned in Section 1, and the throughput sharply decreases when there is an overload. Other reactive-type schemes have been proposed by Lemercier and Pujolle, and by Lakshman et al. [9, 10]. The former discards cells of a packet like PPD, but includes the cells of the same packet that have already been stored in the buffer. In the latter scheme, when a buffer overflow occurs, the cell at the head of the buffer and other cells...
belonging to the same packet are discarded instead of newly arriving packets. Since these two schemes discard cells from the buffer, though, their implementations will be complicated.

A typical preventive-type scheme is early packet discarding (EPD) [2]: a newly arriving packet is rejected if the queue length in the buffer is greater than or equal to a given threshold. The EPD scheme can prevent throughput from decreasing more effectively than PPD. However, as we explain in Section 5, it cannot do this under some traffic conditions such as when an input cell rate, which corresponds to a user cell rate, is much smaller than the output cell rate of the buffer. Other preventive-type schemes have been proposed in [7, 11, 12, 10]. The schemes of [7, 11] accept a newly arriving packet only when the same condition as with EPD is satisfied and the available buffer space is greater than or equal to the size of the packet. This scheme suffers from the same drawbacks as EPD. In the scheme of [12], the threshold value for packet acceptance is changed dynamically depending on the number of packets whose cells are being transmitted through the buffer. In the scheme of [10], newly arriving packets are randomly accepted depending on the queue length in the buffer. With these last two schemes, however, it is difficult to determine the values of their control parameters.

3.2 Proposed scheme

Before describing our proposed scheme, we consider an ideal scheme. Suppose a new packet arrives at time \( t \) and we accept the new packet and reject all packets arriving after time \( t \). We will only accept the packet however, if no buffer overflow occurs after time \( t \). If there will be an overflow, we reject the packet. This ideal scheme is one of the most effective preventive-type schemes that accept packets in the order of arrival. But since the ideal scheme needs complete knowledge of the cell arrival times of the packets that have already been accepted in order to decide acceptability of a newly arriving packet, it seems very difficult to implement. On the other hand, the reported preventive-type schemes mentioned above try to function like the ideal scheme. If all the cells of each packet arrive at one time (batch arrival), though, the whole batch acceptance scheme (WBAS) becomes the ideal scheme. Therefore, we propose a preventive-type scheme in which the behavior of a batch arrival queueing model with a finite capacity buffer is simulated by using the packet-size information contained in the first cells of packets and the WBAS is applied to the model; i.e., a newly arriving packet is accepted only when the available buffer space in the simulated queueing model is greater than or equal to the size of the packet. We call this simulated queueing model a virtual queueing model.

A precise representation of this scheme is given as follows (see Fig. 4). Let \( L_V(t) \) be the length of the virtual queue at time \( t \), where \( L_V(0) = 0 \). \( L_V(t) \) can be implemented by a cell counter in a real system. Let \( K \) be the size of the buffer in the virtual queueing mode, which corresponds to that of the real buffer, and \( W \) be a given threshold used for packet-acceptance decisions. How the value of \( W \) is determined is explained in Subsection 4.3. The packet acceptance decisions are made as follows. Assume that the first cell of a new packet arrives at time \( t \). The packet size \( X_i \) is read from the first cell. If \( K - L_V(t) \geq \max\{W, X_i\} \), the packet is accepted and the value of \( L_V(t) \) is changed to \( L_V(t) + X_i \). If \( K - L_V(t) < \max\{W, X_i\} \), the packet is rejected. The value of \( L_V(t)(>0) \) is decreased one-by-one at intervals of one cell time, even if cells are not actually sent during the intervals; if \( L_V(t) = 0 \), then the value of \( L_V(t) \) remains zero. The cells of an accepted packet enter the buffer if the buffer is not full. All the cells of a rejected packet are discarded, though, even if the buffer is not full.

4 Characteristics of the scheme

4.1 Performance measures

Hereafter, we assume that cell losses occur only at the output buffer in some bottleneck switching node, and call a packet whose cells are successfully transmitted through the system a good packet. The following notations are introduced.

\( \lambda_P \): The mean number of packets arriving during a unit time.

\(^1\)In this case, a buffer overflow may still occur because the cells of packets that have already been accepted before time \( t \) may arrive after time \( t \).

\(^2\)If the available buffer space is greater than or equal to the size of a newly arriving batch, then the batch is accepted. If not, it is rejected.
is the sum of the sizes (the numbers of cells) of the packets arriving at

We consider the following performance measures.

θP: Packet throughput (i.e., the utilization of the output link, = \(a_P m_{\text{acc}}/r_C\)).

fP: A fairness index related to the sizes of the accepted packets\(^3\) (\(= m_{\text{acc}}/m_{\text{all}}\))

In this section, we derive some characteristics of \(\theta_P\) and \(f_P\). A numerical evaluation of \(\theta_P\) and \(f_P\) will be done by computer simulation in Section 5.

### 4.2 Characteristics of packet throughput \(\theta_P\)

The definition of \(\theta_P\) introduced above can be applied in the virtual queueing model; let \(\theta_P^V\) denote the corresponding notation for the virtual queueing model\(^4\). Since only the packets accepted in the virtual queueing model are actually accepted, we obtain

\[
\theta_P^V \geq \theta_P. \tag{1}
\]

This means that the performance of the virtual queueing model is an upper bound of the actual performance. The difference between \(\theta_P^V\) and \(\theta_P\) is caused by the existence of lost cells of packets accepted for the real buffer. We evaluated such cell loss by using a sample path argument \([13]\) without any stochastic assumptions for the packet and cell arrival processes.

Let the unit time be the time needed to send one cell, and assume that all processes begin at time 0. Let \(\mathcal{A}_V \equiv \{(T^V_i, X^V_i)\}\) denote the marked point process that represents the cell arrival process of accepted packets in the virtual queueing model, where \(T^V_i\) is the \(i\)th cell arrival time and \(X^V_i\) is the number of cells arriving together at \(T^V_i\). More than one packet may arrive at \(T^V_i\); in such a case, \(X^V_i\) is the sum of the sizes (the numbers of cells) of the packets arriving at \(T^V_i\). Let \(\mathcal{A} \equiv \{\{T_i, X_i\}\}\) denote the actual cell arrival process of accepted packets. While all the cells of each packet arrive at one time in \(\mathcal{A}_V\), those in \(\mathcal{A}\) may arrive one by one. For each sample path, there exists a unique cell in \(\mathcal{A}\) that corresponds to a cell arriving at \(T^V_i\) in \(\mathcal{A}_V\); letting \(T_j\) denote the arrival time of that cell in \(\mathcal{A}\), we obtain the inequality \(T^V_i \leq T_j\). We also obtain the inequality \(N(t) \leq N^V(t)\), where \(N(t)\) is the number of cells arriving during [0, t] in \(\mathcal{A}\) and \(N^V(t)\) is that in \(\mathcal{A}_V\).

Next, we consider two \(G/D/1/k\) models\(^5\), in which the service times are equal to the unit time. One is a model whose arrival process is \(\mathcal{A}\); its queueing process is denoted by \(\{L^{(k)}(t)\}\). The other is a model

\(^3\)Here we represent the difference between the packet-size distribution of good packets and that of all packets as the proportion of the mean packet size of good packets to that of all packets.

\(^4\)Superscript \(V\) means that the notation is for the virtual queueing model. We use the same expression for other notations.

\(^5\)If an arrival and a departure both occur at the same time, we assume the arrival occurs after the departure.
whose arrival process is $A_V$; its queueing process is denoted by $\{L^{(k)}_V(t)\}$. Since the definition of $A_V$ means that $L^{(k)}_V(t) \leq K$ for any time $t$, the value of $L^{(k)}_V(t)$ does not depend on the value of $k$ if $k \geq K$. Hence, we assume that $k \geq K$ and abbreviate $L^{(k)}_V(t)$ as $L_V(t)$. The following notations are introduced.

$B^{(K)}(t)$: The number of cells that overflow during $[0, t]$ in the $G/D/1/K$ model whose arrival process is $A$.

$B_n^{(K)}$: The number of cells that overflow during the $n$th busy period in the same $G/D/1/K$ model.

$\gamma^{(K)}_n$: The beginning time of the $n$th busy period in the same $G/D/1/K$ model.

$D_V(t)$: The sum of service times completed during $[0, t]$ in the $G/D/1/K$ model whose arrival process is $A_V$.

$D^{(\infty)}(t)$: The sum of service times completed during $[0, t]$ in the $G/D/1/\infty$ model whose arrival process is $A$.

The inequality $N(t) \leq N_V(t)$ leads to the inequality $D^{(\infty)}(t) \leq D_V(t)$. Using these definitions, we obtain the following theorem. (The proof is in the appendix.)

**Theorem 1** For each sample path,

$$B_n^{(K)} \leq \left\lfloor D_V(\gamma^{(K)}_n) - D^{(\infty)}(\gamma^{(K)}_n) \right\rfloor, \quad n \geq 1,$$

where $\lceil x \rceil$ is the maximal integer that is not greater than $x$.

From this theorem, the following corollary is derived.

**Corollary 1** For any sample path and any time $t$ such that $\gamma^{(K)}_n \leq t < \gamma^{(K)}_{n+1}$,

$$B^{(K)}(t) \leq \sum_{j=1}^{n} \left[ D_V(\gamma^{(K)}_j) - D^{(\infty)}(\gamma^{(K)}_j) \right].$$

If the cell arrival interval of each packet is less than or equal to the unit time, the busy periods of $\{L_V(t)\}$ coincide with those of $\{L^{(\infty)}(t)\}$ and the equality $D_V(t) = D^{(\infty)}(t)$ is obtained for any time $t$. From this and Corollary 1, the value of $B^{(K)}(t)$ becomes zero, and we obtain the following corollary.

**Corollary 2** Assume that the cell arrival intervals of each packet are less than or equal to the unit time with probability one. If the processes are ergodic, then

$$\theta^V_p = \theta_p.$$

If the cell arrival interval of each packet is less than or equal to the unit time, the busy periods of $\{L_V(t)\}$ coincide with those of $\{L^{(\infty)}(t)\}$ and the equality $D_V(t) = D^{(\infty)}(t)$ is obtained for any time $t$. From this and Corollary 1, the value of $B^{(K)}(t)$ becomes zero, and we obtain the following corollary.

**Corollary 3** For any $n \geq 1$,

$$B_n^{(K)} \leq g_{\text{max}} \quad \text{w.p. 1}.$$
Corollary 3 means that the number of cells that overflow from a real buffer during a busy period is less than or equal to the value that can be determined independently of the value of $\rho_C$ even if it is very high. Hence, for a sufficiently large $K$, we expect the relation $\theta_P \approx \theta_P$ to be asymptotically satisfied as the value of $\rho_C$ increases. This is one reason why our scheme can prevent throughput decreasing when there is an overload.

In this section, we have estimated the performance of our scheme in comparison with that of the virtual queueing model, because we expect the performance of the virtual queueing model to give a sufficiently good approximation of the performance of the ideal scheme. Of course, a scheme that can achieve better performance than that of the virtual queueing model may exist.

### 4.3 Characteristics of the fairness index $f_P$

As we will show in Section 5, when the value of $W$ is set to zero, our scheme is apt to accept smaller packets in an overload situation. We explain this as follows. If $W = 0$ and the available buffer space of the virtual queueing model is greater than or equal to the size of a newly arriving packet, the packet is always accepted. Hence, in an overload situation, our scheme ($W = 0$) may continually accept smaller packets arriving before the available buffer space becomes large enough to accept a larger packet. To overcome this problem, we added a trunk reservation scheme, which equalizes the loss probabilities of integrated-hybrid-traffic [14], to our scheme, and set the value of $W$ to the maximal packet size. We expect this to ensure that the relation $f_P \approx 1$ is satisfied. Especially, from the PASTA (Poisson arrival sees time averages) property [15], we can see that if packets arrive according to a Poisson process, the size distribution of the accepted packets should coincide with that of all packets.

If the value of $W$ is too large, however, the real buffer will not be used effectively. Hence, we propose that it is set to the maximum size of the packets that should be accepted fairly.

### 5 Numerical evaluation

In this section, we compare our scheme with the PPD and EPD schemes through a computer simulation that uses a Monte Carlo method and actual LAN traffic data to generate packet arrivals at the cell buffer. In [16, 17], the behavior of a selective cell discarding scheme was represented as a Markovian model, and some performance measures were obtained. However, it is difficult to apply such an analysis method to our scheme because the number of states in the Markovian model becomes too large.

#### 5.1 Simulation model

We used the output buffer model shown in Fig. 3 as a simulation model. While the packet arrival process at the output buffer consists of packet flows from many users, we took no account of each user in the simulation model. We also did not assume any packet flow control on the upper layers. All cell losses are caused by selective cell discarding or buffer overflow. Let the input cell rate into the buffer be the peak cell rate of the output links of users. The cells of a packet arrive according to this input cell rate; the interarrival times of the consecutive cells of a packet are set to $1/(\text{input cell rate})$ with variations uniformly distributed in the 10% value of $1/(\text{input cell rate})$. These variations represent the cell-delay variations in the network. The output cell rate from the buffer was set to the peak cell rate of the output link connected to the buffer. In our numerical experiments, the ratio of the input cell rate to the output cell rate was set to $1/1$ or $1/25$. For example, the latter ratio corresponds to a case where the value of the input cell rate is $(6 \times 10^6) \text{ [bps]}/(53 \times 8) \text{ [bits]}$ and that of the output cell rate is $(150 \times 10^6) \text{ [bps]}/(53 \times 8) \text{ [bits]}$. Assuming the LANs connected to the network are Ethernets, the minimum and maximum frame sizes in the Ethernets are 64 bytes and 1518 bytes, respectively. Adding overheads of the CLNAP and the AAL type 3/4 to the frames, the minimum and maximum packet sizes corresponding to the frames are 3 cells and 36 cells. Taking these packet sizes into account, we let the value of the threshold $W$ be 36 cells for the EPD and be 0 or 36 cells for our scheme.

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To put it more precisely, we need the condition that the length of a busy period in the $G/D/1/K$ model whose arrival process is $A$ becomes sufficiently large as the value of $\rho_C$ increases.
5.2 Results of the Monte Carlo simulations

In the Monte Carlo simulations, we assumed that packets arrive according to a Poisson process. The buffer size was 360 cells. Figure 5 shows the values of $\theta_P$ and $f_P$ in the case where packet sizes are uniformly distributed between 3 cells and 36 cells. Since the length of the 95% confidence interval of each value is very small\(^7\), we omitted the intervals from all figures. From these results, we can see that the values of $\theta_P$ for the PPD and the EPD decrease when there is an overload, especially for the case where the proportion of the input cell rate to the output cell rate is small ($= 1/25$). We attribute this to the following. For the PPD, resources such as the buffer space and output link are wasted by useless cells that enter the buffer before the buffer overflow occurs. Furthermore, when the buffer is not full, a newly arriving packet is accepted even if the cells of many packets are simultaneously arriving at the buffer, and this tends to quickly fill the buffer. For the EPD, too many packets may be accepted before the queue length in the buffer exceeds the threshold, and this causes buffer overflows and decreases the value of $\theta_P$.

While the values of $f_P$ for the EPD and our scheme ($W = 36$) remain close to one, those of the other schemes decrease during an overload. This is because with PPD larger size packets are apt to experience cell loss caused by buffer overflow during an overload. With our scheme ($W = 0$), a smaller packet is likely to be accepted before the available buffer space of the virtual queuing model becomes large enough to accept a larger packet.

Figure 6 shows the values of $\theta_P$ when the packet size was constant at 36 cells. As in Fig. 5, the values of $\theta_P$ for PPD and EPD decrease when there is an overload. Again, our scheme performs well.

5.3 Results in measured data simulations

In recent years, papers have reported that LAN traffic has an unusually bursty nature that is called self-similarity [18], and the above Poisson process that we used to simulate packet arrival is not as bursty as LAN traffic. Thus, we repeated the simulation using actual LAN traffic data to generate packet arrivals. This LAN traffic data was measured at the Bellcore Morristown Research and Engineering Center, and includes one million packets arriving from outside sources (the data is available at http://ita.ee.lbl.gov/index.html). In the simulations, CLNAP and AAL type 3/4 headers and trailers were added to each packet, and the packets were divided into cells. Since the LANs where the traffic was measured were Ethernets, the minimum and maximum sizes of the packets were 3 cells and 36 cells, respectively. The value of the offered traffic was controlled by changing the value of the time needed to send one cell. The buffer size $K$ was set at 1080 cells, which is thirty times as large as the maximum packet size. Figure 7 shows the simulation results. Since the LAN traffic was very bursty, many packets were discarded even when the offered traffic was low, and the characteristics of the curves of $\theta_P$ and $f_P$ are very different from those in Figs. 5 and 6. With respect to $\theta_P$, our scheme is best; however, the differences between the values of $f_P$ are small. The LAN traffic was measured over about one hour, though, and we think almost all packet losses occurred during short congestion periods during the one hour. Thus, the differences between the $f_P$ values during the actual congestion periods would be rather large. With respect to $f_P$, our scheme ($W = 36$) was best; however, for all the schemes, the $f_P$ values decreased as the offered traffic increased. We conjecture that this is because the packet sizes varied depending on the offered traffic; i.e., they became larger as the offered traffic became higher. We also did simulation experiments for other cases buffer sizes and obtained similar results.

6 Conclusions

We have proposed a selective cell discarding scheme that uses packet-size information, and have shown by computer simulation that our scheme can prevent decreased packet throughput even when the traffic load is very high and can also accept packets fairly, regardless of the packet sizes.

\(^7\)For example, in Figs. 5(a) and (c), when $\rho = 1.0$, the 95% confidence interval for $\theta_P = 0.96641$ is 0.00107 and that for $f_P = 19.505$ is 0.034 with our scheme ($W = 36$).
References


Appendix: Proof of Theorem 1

First, we introduce some notations and derive a lemma. Let \( \{M^{(k)}(t)\} \) denote the unfinished work in a \( G/D/1/k \) model whose arrival process is \( A \), and let \( \{M^{(k)}_{\nu}(t)\} \) denote that in another \( G/D/1/k \) model.
whose arrival process is \( A_V \). Assume that \( k \geq K \), and abbreviate \( M^{(k)}_V(t) \) as \( M_V(t) \) like \( L^{(k)}_V(t) \). From the Lindley formula, we obtain

\[
\begin{align*}
M_V(T_i^V) &= X_i^V, \\
M_V(T_{i+1}^V) &= [M_V(T_i^V) - (T_{i+1}^V - T_i^V)]^+ + X_{i+1}^V, \quad i \geq 2, \\
M^{(k)}_V(T_1) &= X_1 - O^{(k)}(T_1), \\
M^{(k)}_V(T_{i+1}) &= [M^{(k)}_V(T_i) - (T_{i+1} - T_i)]^+ + X_{i+1} - O^{(k)}(T_{i+1}), \quad i \geq 2,
\end{align*}
\]

where \( O^{(k)}(T_i) \) is the number of cells that overflow at time \( T_i \). It is given as

\[
O^{(k)}(T_{i+1}) = \left[ \left( M^{(k)}(T_i) - (T_{i+1} - T_i) \right)^+ + X_{i+1} - K \right]^+, \quad i \geq 1,
\]

where \( [x]^+ \equiv \max\{0, x\} \). For any time \( t \), we obtain the following.

\[
\begin{align*}
M_V(t) &= [M_V(T_i^V) - (t - T_i^V)]^+, \quad T_i^V \leq t < T_{i+1}^V, \\
M^{(k)}(t) &= [M^{(k)}(T_i) - (t - T_i)]^+, \quad T_i \leq t < T_{i+1}.
\end{align*}
\]

The numbers of cells in the systems are given as \( L_V(t) = [M_V(t)] \) and \( L^{(k)}(t) = [M^{(k)}(t)] \), respectively. Hereafter, we assume that \( \delta(t) \equiv D_V(t) - D^{(\infty)}(t) \). The lemma is given as follows.

**Lemma 1** For any time \( t \geq 0 \),

\[
N_V(t^-) - N(t^-) \leq \delta(t) + M_V(t^-). \tag{6}
\]

(Proof) If \( k = \infty \), then buffer overflows never occur. From this, we obtain that \( N^{(\infty)}(t^-) = D^{(\infty)}(t^-) + M^{(\infty)}(t^-) \). Furthermore, we obtain the relation \( N_V(t^-) = D_V(t^-) + M_V(t^-) \). From these formulas, the result is derived taking account of the relations \( D_V(t^-) = D_V(t), D^{(\infty)}(t^-) = D^{(\infty)}(t) \), and \( M^{(\infty)}(t^-) \geq 0 \).

(Proof of Theorem 1)

Let \( \{(T_{i+\ell}, X_{i+\ell})\}_{i=1}^j \) denote the arrival process during the \( n \)th busy period in a \( G/D/1/K \) model whose arrival process is \( A_V \). Separate this arrival process into two parts: one is \( \{(T_{i+\ell}, X^{(1)}_{i+\ell})\}_{i=1}^j \) and the other is \( \{(T_{i+\ell}, X^{(2)}_{i+\ell})\}_{i=1}^j \). The former contains cells arriving at or after time \( T_{i+1} \) in process \( A_V \), and the latter contains cells arriving before time \( T_{i+1} \). From these definitions, we obtain the relation \( X^{(1)}_{i+\ell} + X^{(2)}_{i+\ell} = X_{i+\ell} \) for \( 1 \leq \ell \leq j \). Denoting the cell arrival process during \( [T_{i+1}, T_{i+k}] \) in \( A_V \) as \( \{(T^V_{i+\ell}, X^V_{i+\ell})\}_{\ell=1}^{k'} \) for \( 1 \leq k \leq j \), we obtain the inequality \( \sum_{\ell=1}^{k} X^{(1)}_{i+\ell} \leq \sum_{\ell=1}^{k'} X^V_{i+\ell} \). From Lemma 1, we further obtain

\[
\sum_{\ell=1}^{k} X^{(2)}_{i+\ell} \leq \delta(T_{i+1}) + M_V(T_{i+1}^-), \quad 1 \leq k \leq j.
\]

Since there exists a busy period of \( \{M^{(K)}(t)\} \) that contains the interval \( [T_{i+1}, T_{i+j}] \), we obtain

\[
\begin{align*}
&\quad \left( M^{(K)}(T_{i+k+1}) - (T_{i+k} - T_{i+k-1}) \right)^+ + X_{i+k} \\
&= \left( M^{(K)}(T_{i+k}) - (T_{i+k} - T_{i+k-1}) \right)^+ + X_{i+k} \\
&= \sum_{\ell=1}^{k} X_{i+k} - (T_{i+k} - T_{i+1}) - \sum_{\ell=1}^{k-1} O^{(K)}(T_{i+\ell}) \\
&\leq \delta(T_{i+1}) - \sum_{\ell=1}^{k-1} O^{(K)}(T_{i+\ell}) + M_V(T_{i+1}^-) + \sum_{\ell=1}^{k'} X^V_{i+\ell} - (T_{i+k} - T_{i+1}) \\
&\leq \delta(T_{i+1}) - \sum_{\ell=1}^{k-1} O^{(K)}(T_{i+\ell}) + M_V(T_{i+k}). \tag{8}
\end{align*}
\]
From this formula and $M_V(t) \leq K$ for $t \geq 0$, we obtain

$$O^{(K)}(T_{i+k}) = \left\lceil \left[ [M^{(K)}(T_{i+k-1}) - (T_{i+k} - T_{i+k-1})]^+ + X_{i+k} - K \right]^+ \right\rceil$$

$$\leq \left\lceil \left[ \delta(T_{i+1}) - \sum_{\ell=1}^{k-1} O^{(K)}(T_{i+\ell}) \right]^+ \right\rceil$$

for $1 \leq k \leq j$. \hspace{1cm} (9)

From this, the following formula is derived.

$$\sum_{\ell=1}^{k} O^{(K)}(T_{i+\ell}) \leq \lceil \delta(T_{i+1}) \rceil \text{ for } 1 \leq k \leq j. \hspace{1cm} (10)$$

From this formula and the relations $B^{(K)}_n = \sum_{\ell=1}^{j} O^{(K)}(T_{i+\ell})$ and $T_{i+1} = \gamma^{(K)}_n$, Eq. (2) is obtained. \hspace{1cm} \Box
Figure 5: Performance comparison by the Monte Carlo method (Packet size distribution is U[3, 36], $K = 360$).
Figure 6: Performance comparison by the Monte Carlo method (Constant packet size (= 36), $K = 360$).
Figure 7: Performance comparison using actual data ($K = 1080$).