No. 1

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Essential state surfaces for knots and links

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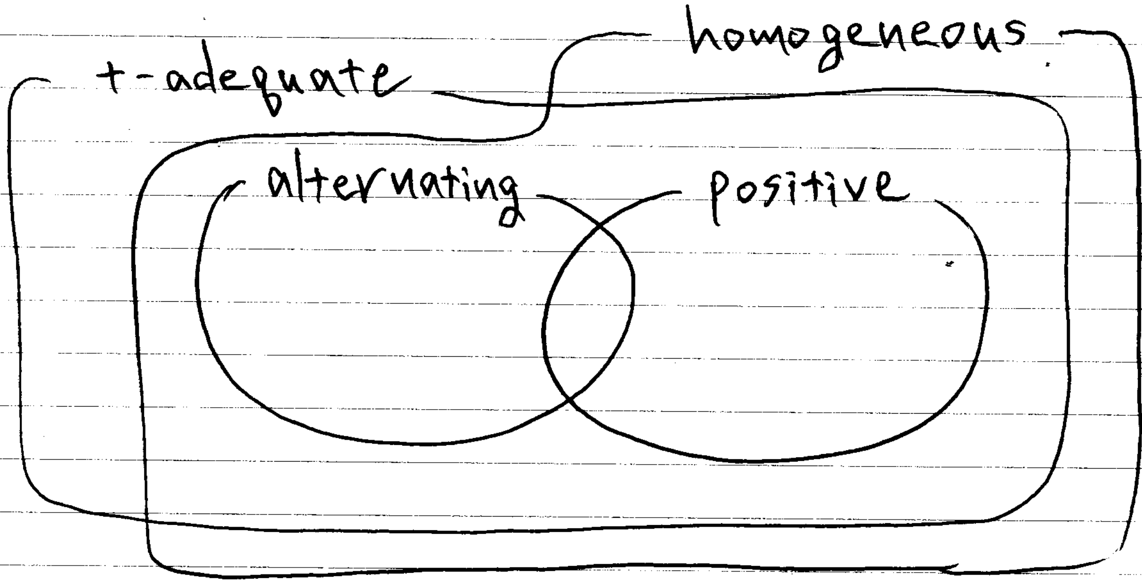
We introduce a new class, which contains

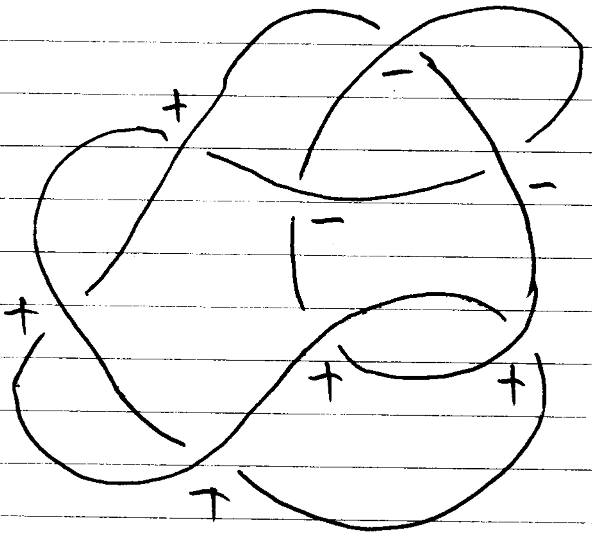


and show that these knots and links bound essential state surfaces.

As applications, using essential state surfaces we solve the determining problem for splittability, triviality and primeness

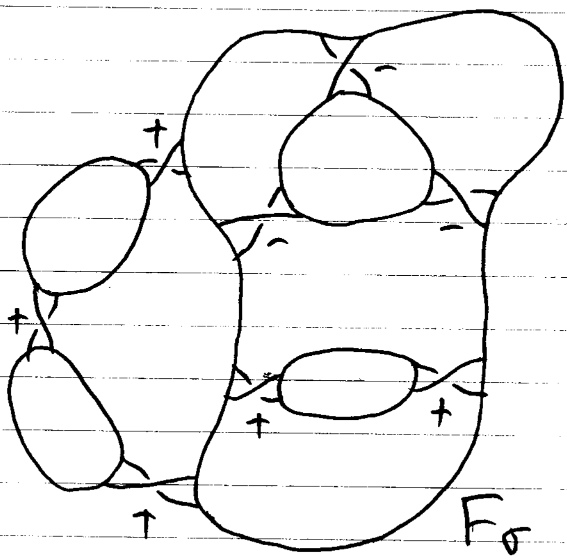
State surface and spine graph

σ : {crossings} $\to$ {+, −} ; state

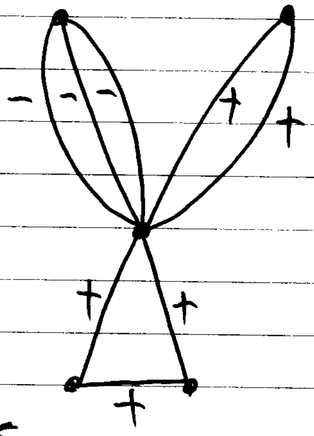


D

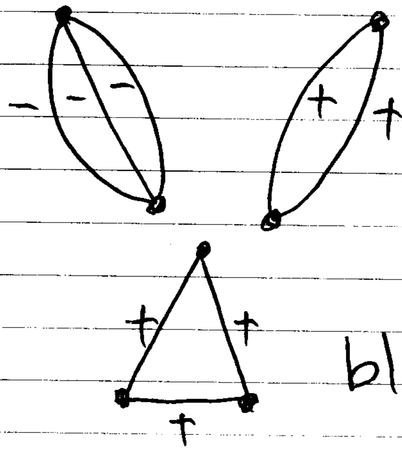
$\Rightarrow$



F_σ



G_σ



blocks

Definition

D is σ-adequate $\Leftrightarrow$ $\nexists$ loop in G_σ

D is σ-homogeneous $\Leftrightarrow$ All edges in each block have a same sign

Theorem 1

D ; σ-adequate and σ-homogeneous
$\Rightarrow F_\sigma$ is π_1-essential

Theorem 2

Let K be a knot or link which admits a σ-adequate and σ-homogeneous diagram D without nugatory crossings. Then,

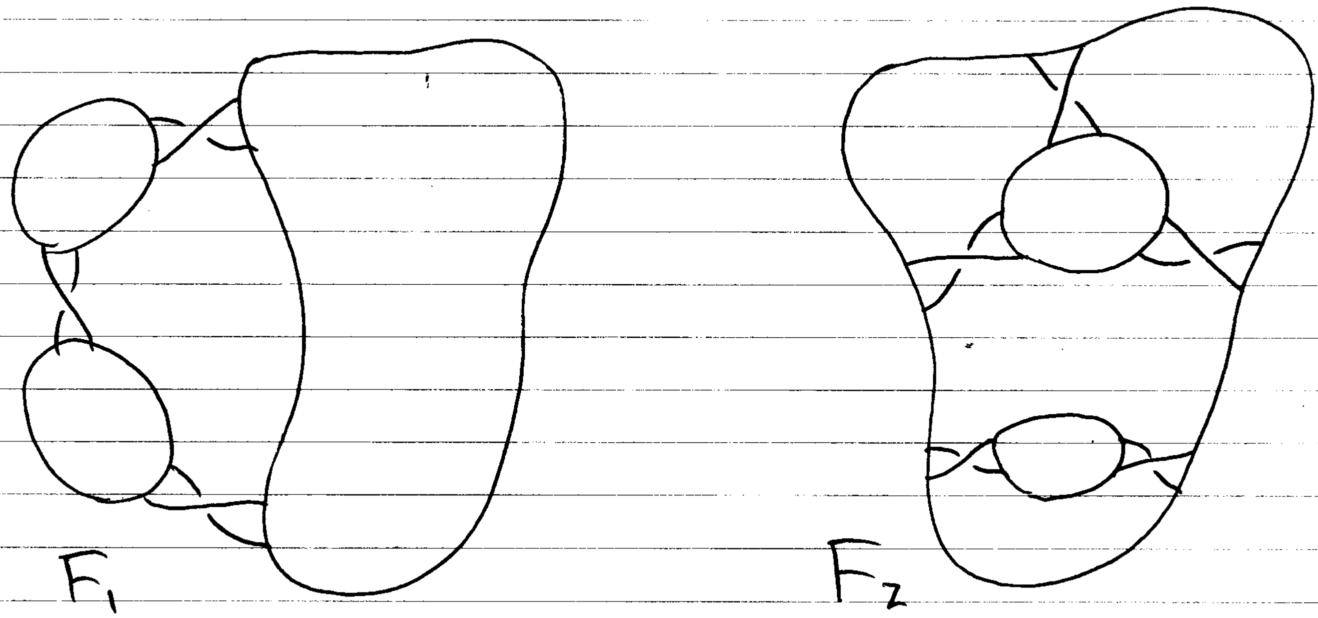
(1) K is non-split if and only if D is connected.

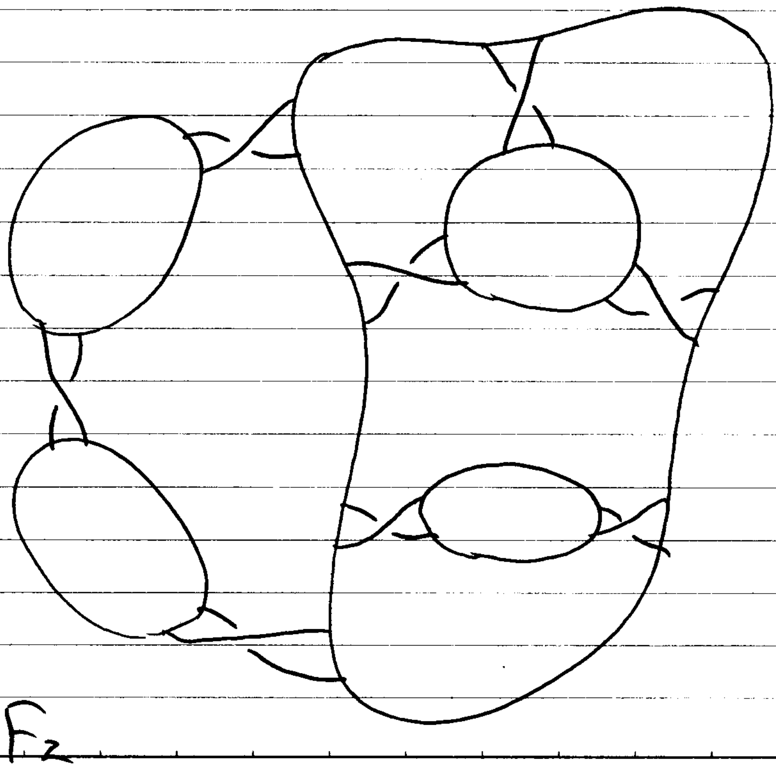
(2) K is non-trivial if and only if D has at least one crossing.

(3) K is prime if and only if D is prime.

Key Lemma 3

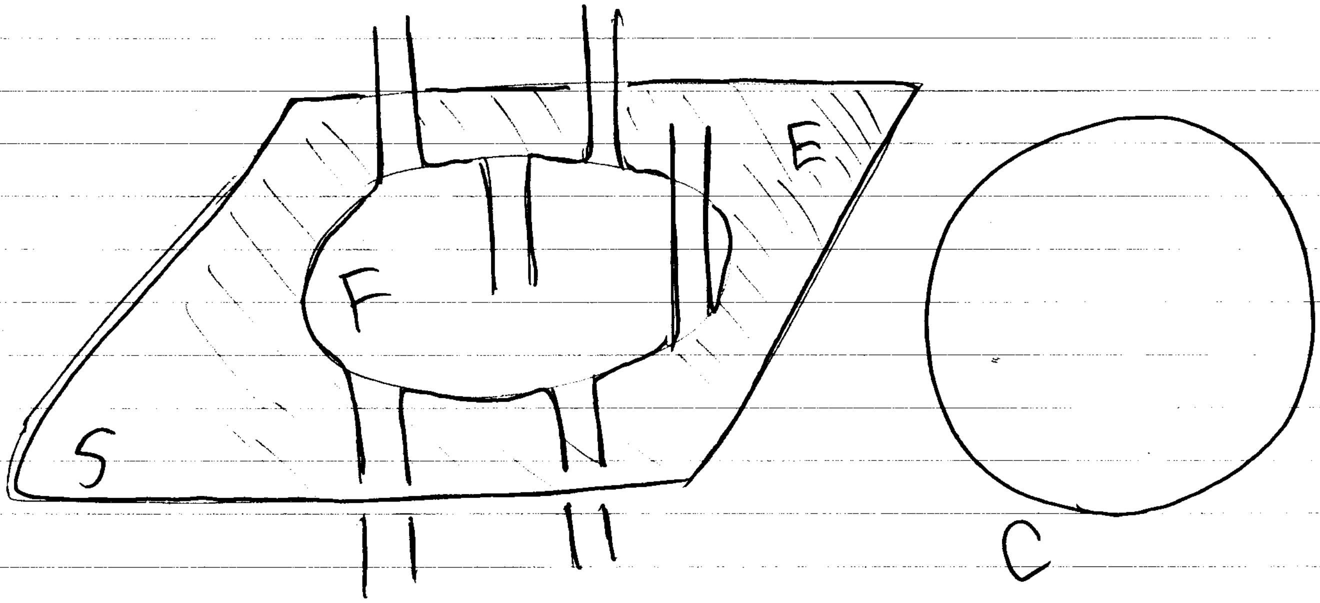
F_1, F_2 ; π_1-essential
$\Rightarrow F_1 * F_2$; π_1-essential





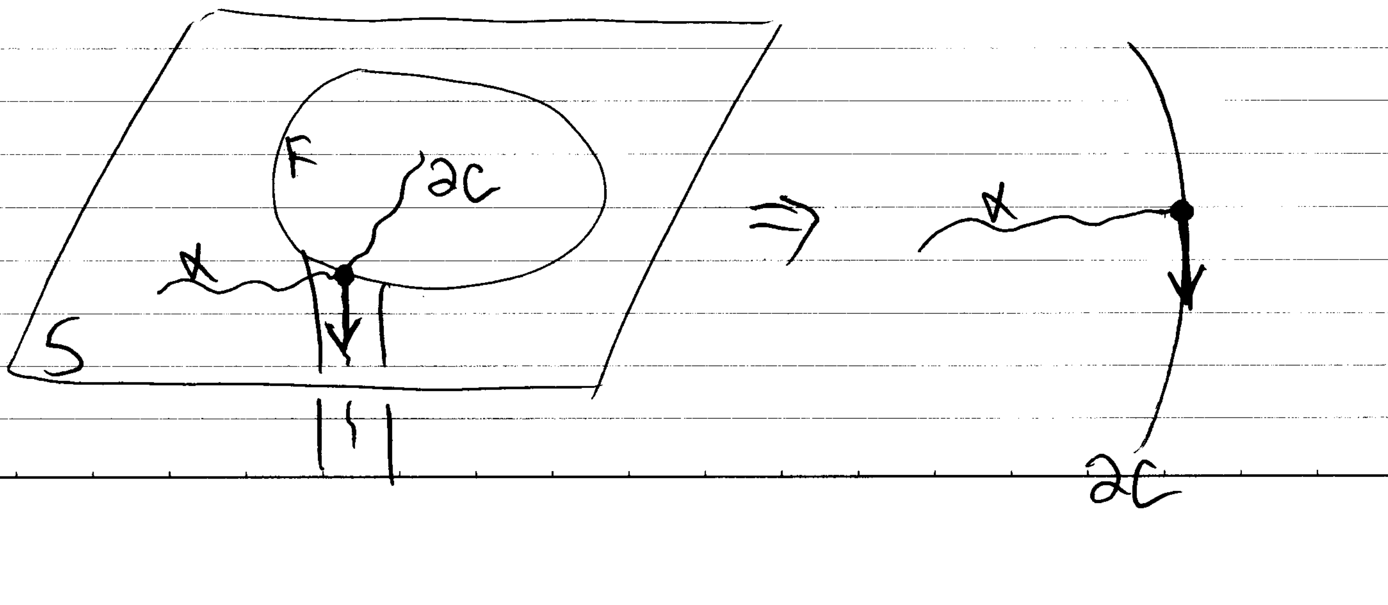
Proof. Suppose $F = F_1 * F_2$ has a compressing disk C.

Let S be the decomposing sphere for $F_1 * F_2$



Put $E = cl(S - F)$ and focus on $C \cap E$

Mark each point of $\partial(C \cap E)$ on ∂C by an arrow from $F \cap S$ to $F - S$

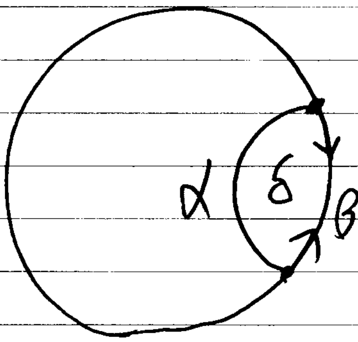


Claim

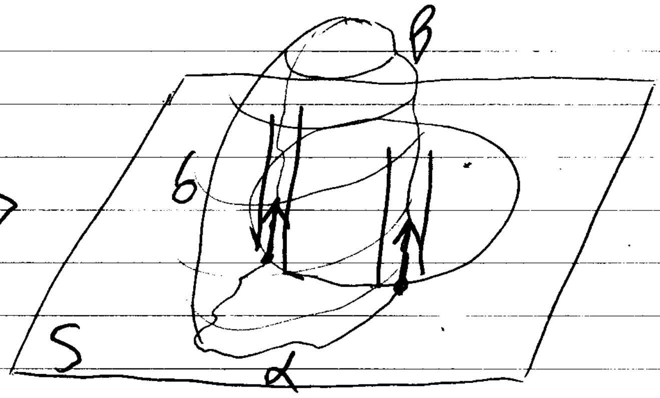
> Every arc of $C \cap E$ has a compatible arrows at its boundary, and each subdisk of $C - E$ has a source.

(proof) For an outermost arc α,

If

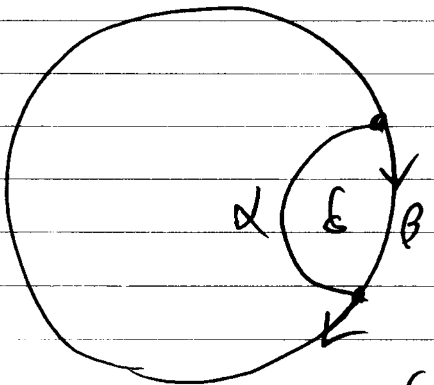


$\Rightarrow$

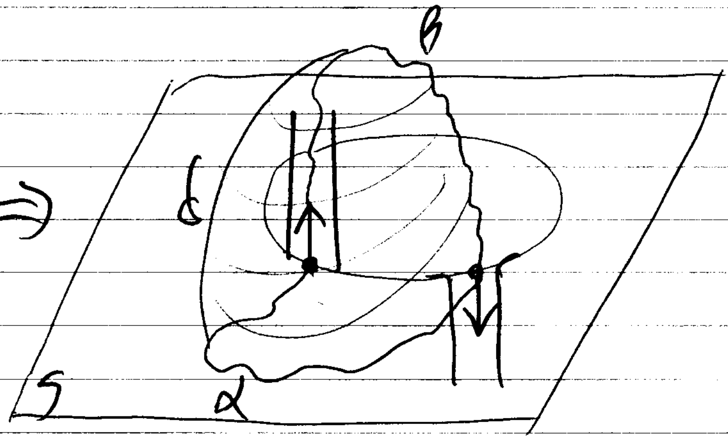


δ can be extended to a compressing disk for F_1

If

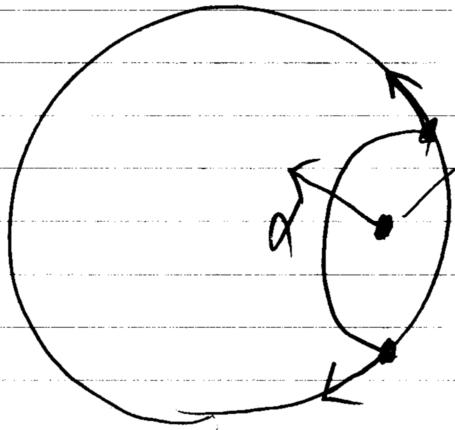


$\Rightarrow$

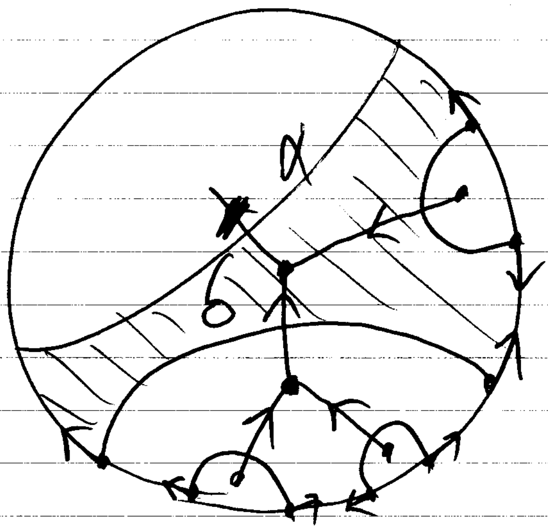


δ can be extended to a ∂-compressing disk for F_1

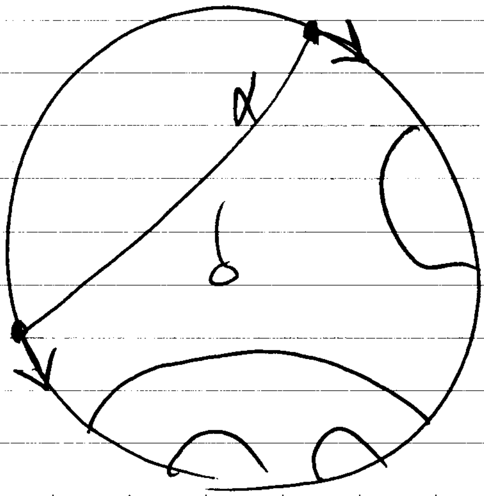
Hence



By an induction,

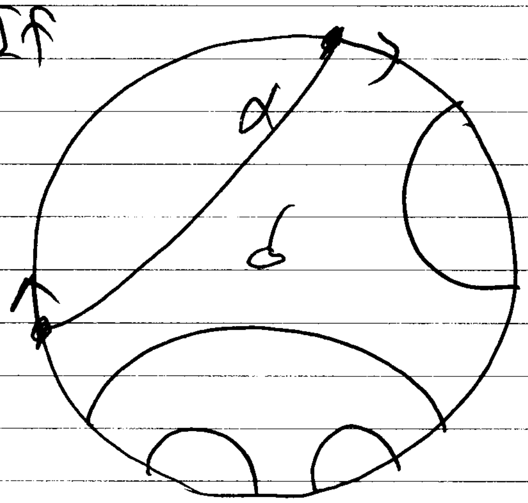


If



$\Rightarrow$ δ can be extended t
a compressing disk for F

If



$\Rightarrow$ δ can be extended to a ∂-compressing disk for F_i

$\blacksquare$ (claim)

By claim, the dual graph on C has an oriented cycle.

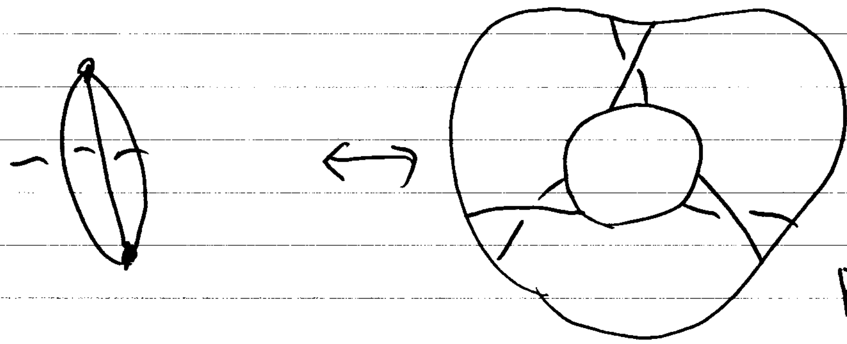
This is impossible because the graph is tree.

$\blacksquare$ (Key Lemma)

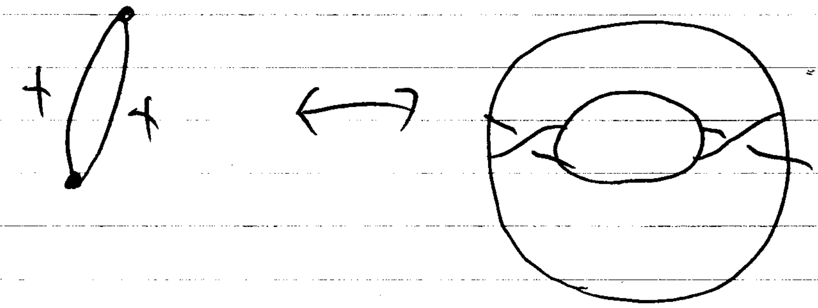
Lemma 4 (Menasco - Thistlethwaite) 1993.

> D ; reduced, prime, alternating diagram
> $\Rightarrow$ checkerboard (coloring) surface is π_1-essential.

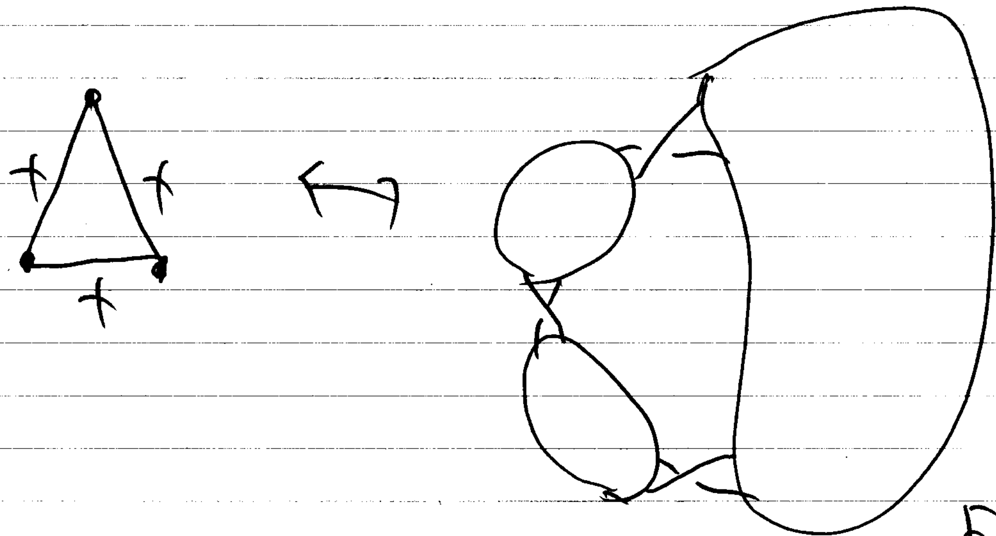
By Lemma 4, every surfaces corresponding each block of G_σ are π_1-essential.



F_1



F_2



F_3

By Lemma 3, F_σ is π_1-essential.

$F_\sigma = F_3 * (F_1 * F_2)$